SHENTON COLLEGE

ATMAS Mathematics Specialist
2019 Test 2 Calculator Free

Name: ……

Time Allowed : 50 minutes

Marks /59

Materials allowed: No special materials.

All necessary working and reasoning must be shown for full marks.
Where appropriate, answers should be given in exact values.
Marks may not be awarded for untidy or poorly arranged work.

1 For a line passing through the point $\begin{pmatrix} 5 \\ -1 \end{pmatrix}$ and parallel to the vector $\begin{pmatrix} 1 \\ 4 \end{pmatrix}$, find

a) The vector equation of the line. (1)

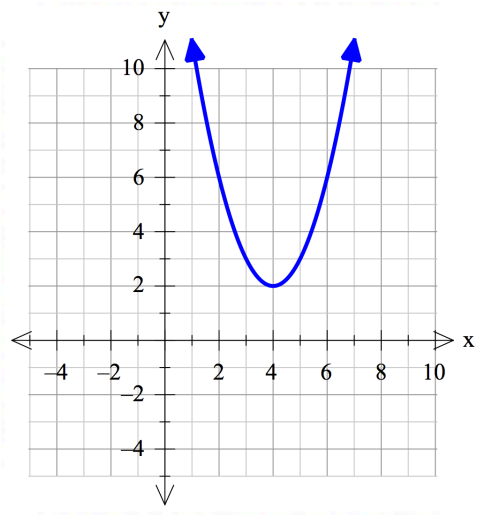
b) The parametric equations of the line. (2)

c) The Cartesian equation of the line. (2)

2 Line L_1 has the vector equation $\mathbf{r} = \begin{pmatrix} 2 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 3 \end{pmatrix}$. Find the equation of L_2, a line perpendicular to L_1 and passing through position $\begin{pmatrix} 1 \\ -5 \end{pmatrix}$. (2)

3 If $f(x) = 9 - x^2$ and $g(x) = \sqrt{x+7}$, determine the domain and range of the composition $g(f(x))$. (4)

4 The graph below shows the function $f(x) = (x-4)^2 + 2$.



a) Determine an appropriate restriction on the domain of $f(x)$ so that the inverse $f^{-1}(x)$ exists and is a decreasing function. (1)

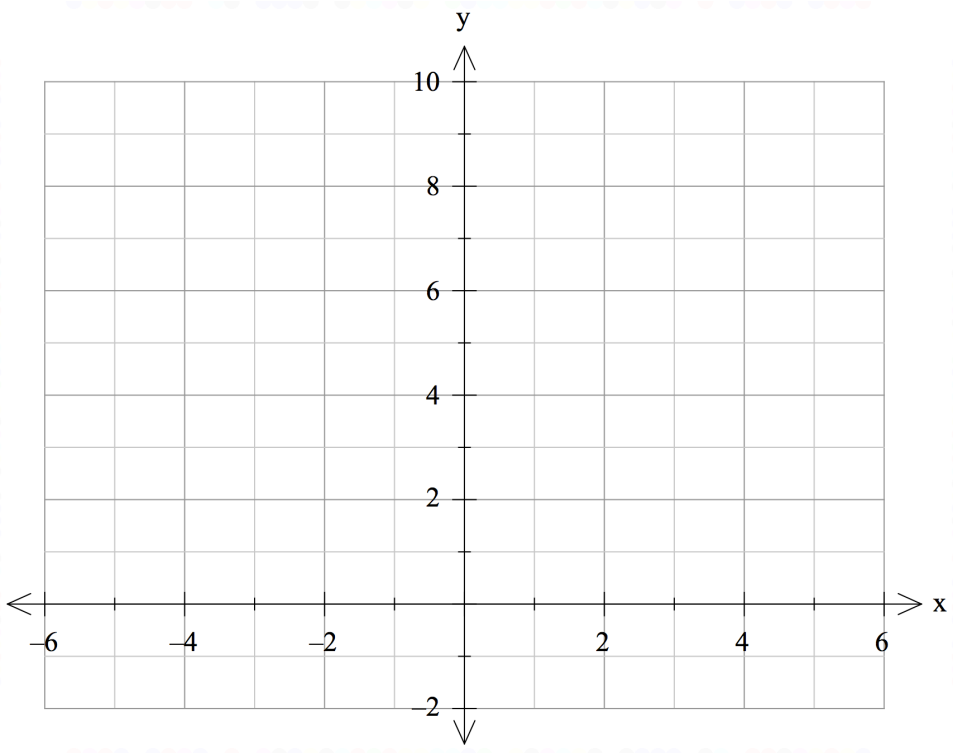
b) Give the equation of the inverse based on your answer to part (a) in the form $y = \ldots$ (2)

5 The function f(x) is defined as $f(x) = |x + 3| + |x - 1|$

a) Remove the absolute value signs by writing the function in piecewise form. (3)

$$f(x) = \begin{cases} \_\_\_\_\_ \\ \_\_\_\_\_ \\ \_\_\_\_\_ \end{cases}$$

b) Sketch the function $f(x) = |x + 3| + |x - 1|$ on the set of axes below. (3)



c) Hence of otherwise solve $|x + 3| + |x - 1| = 8$ (1)

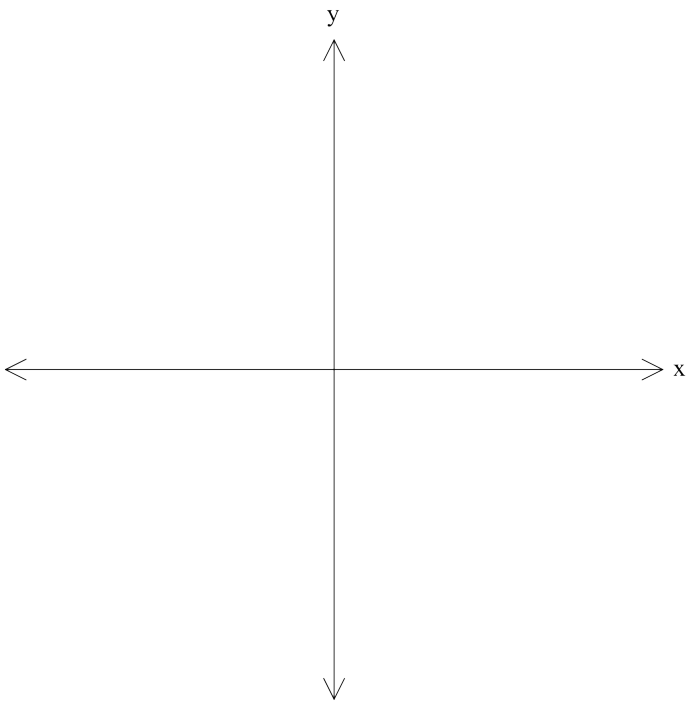
6 The position vectors $\begin{pmatrix}5\\2\\2\end{pmatrix}$, $\begin{pmatrix}-3\\4\\-1\end{pmatrix}$ and $\begin{pmatrix}2\\6\\-5\end{pmatrix}$ are all points on the plane P_1.

a) Determine the vector equation of P_1 using appropriate parameters. (3)

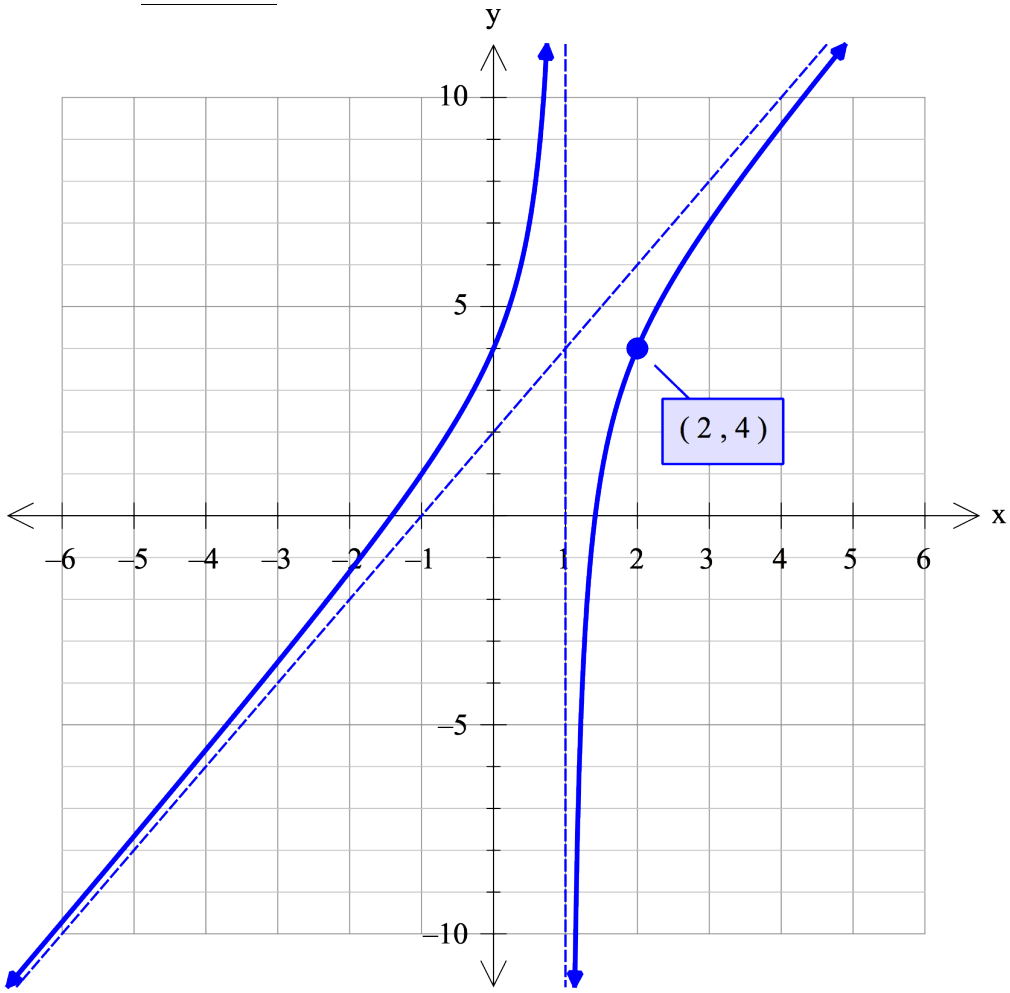
b) Determine the Cartesian equation of P_1. (4)

7 If $h(x) = \frac{1}{8^x}$ and $h\big(k(x)\big) = 2^{3-3x-3x^2}$, find the equation of $k(x)$. (4)

8 Consider the rational function $y = \frac{3x^2}{x-1}$. Given that $\frac{d^2y}{dx^2} = \frac{6}{(x-1)^3}$, draw a sketch of the function, indicating on your sketch important features such as asymptotes, intercepts, and critical points. (6)

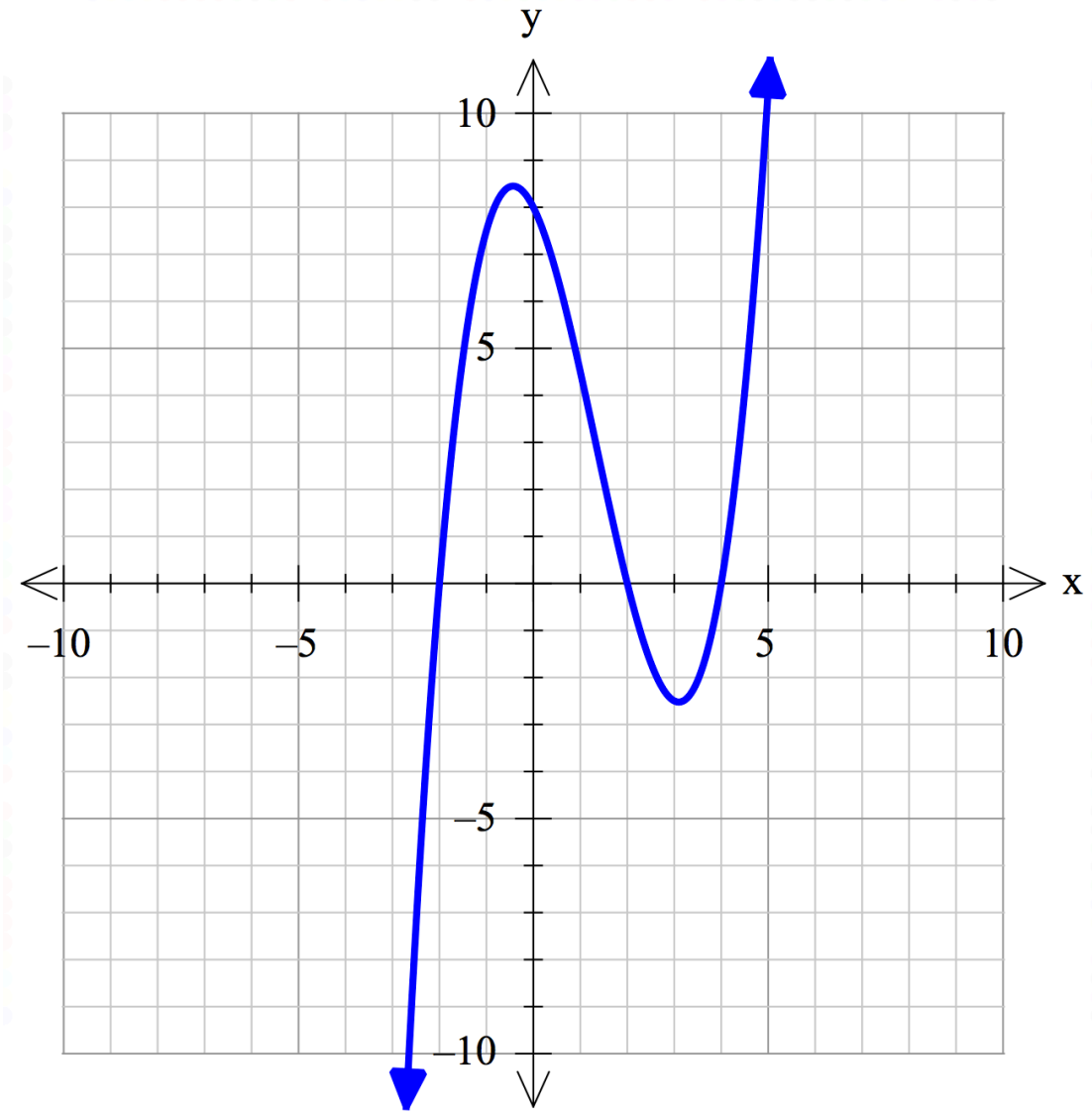


9 The graph below was created from the function $y = \frac{f(x)}{g(x)}$. Both $f(x)$ and $g(x)$ are functions with order less than 3.



Determine both $f(x)$ and $g(x)$. (5)

10 The graph below shows $y = f(x)$.



a) Mark on the graph the points where $f(x)$ would intersect with $\frac{1}{f(x)}$. (2)
(Do not graph $\frac{1}{f(x)}$.)

b) Add a sketch of $f^{-1}(x)$ to the axes above, indicating at least 3 key points. (4)

c) Explain why $f^{-1}(x)$ is not a function. (1)

11

a) Determine any points of intersection between the sphere $\left|\boldsymbol{r}-\begin{pmatrix}5\\-1\\2\end{pmatrix}\right|=3$ and the line $\boldsymbol{r}=\begin{pmatrix}-2\\3\\2\end{pmatrix}+\lambda\begin{pmatrix}1\\-1\\0\end{pmatrix}$ (4)

b) Calculate the shortest distance between the sphere $\left|\boldsymbol{r}-\begin{pmatrix}5\\-1\\2\end{pmatrix}\right|=3$ and the plane $r\cdot\begin{pmatrix}3\\0\\-4\end{pmatrix}=-68$ (5)